Endogenous Comparative Advantage[*]

Andrea Moro[§] Peter Norman[¶]

February 14, 2015

Abstract

We develop a stylized model of trade between identical countries. The departure from neoclassical theory is that endogenous human capital investments are imperfectly observed. Investments have a public good component: firms use aggregate country investment as the prior when evaluating workers, which creates an informational externality. The interaction between this externality and general equilibrium effects creates asymmetric equilibria with comparative advantages even when there is a unique equilibrium under autarky. Symmetric, no-trade equilibria may be unstable under free trade. Welfare effects are ambiguous: trade may be Pareto improving even if it leads to an asymmetric equilibrium with rich and poor countries.

Keywords: Comparative Advantage, Specialization, Human Capital.
JEL Classification Number: D62, D82, F11, O12

1 Introduction

In this paper we develop a stylized model of international trade in which the interaction between general equilibrium effects and an informational externality generates a novel explanation for specialization and trade. Competitive firms in two ex ante identical countries have access to the same technology. Two goods are produced using two types of labor input:

[*]This paper has circulated for so long that is impossible to thank everybody that contributed comments and suggestions. We thank them all. Special thanks to Lutz Hendricks, Pat Kehoe, Tim Kehoe, Narayana Kocherlakota, John Knowles, Rody Manuelli, Andrew Postlewaite, Paul Segerstrom, Ananth Seshadri, Robert Staiger, Kjetil Storesletten, Scott Taylor and Fabrizio Zilibotti for helpful comments, suggestions and discussions. Support from NSF Grants #SES-0003520 and #SES-0001717 is gratefully acknowledged. The research for this paper was done partly when Peter Norman was visiting IIES Stockholm and he is grateful for their hospitality.

[§]Department of Economics, Vanderbilt University and Dipartimento di Economia, Universitàa Ca' Foscari di Venezia. Email: andrea@andreamoro.net

[¶]Department of Economics, University of North Carolina at Chapel Hill. Email: normanp@email.unc.edu

skilled and *unskilled* labor. A "high tech" good can be produced from skilled labor only, whereas skilled and unskilled labor are perfect substitutes in the production of a "low-tech" good.[1] Trade is frictionless, but there is an informational friction in the labor market.

Two crucial assumptions drive our results: 1) workers acquire their skills from costly human capital investments, and 2) firms observe a noisy measure of skills. These two assumptions generate an informational externality because the wage distribution in a country depends on beliefs about human capital investments, which in a rational expectations equilibrium is determined by average human capital investments. This implies that investments in human capital have a "public good component": higher average human capital investments increase expected wages for all workers, regardless of their human capital.

Conditional on human capital investments in the two countries, the model is like a Hecksher-Ohlin model, except that individual wages in the high-tech sector are higher for workers that are more likely to be skilled. Firms estimate workers' productivity by Bayesian updating, using a noisy signal of skill and a prior that is determined by average investment, which in our setup is the equilibrium proportion of skilled workers.

This generates a wage schedule where workers drawing a high signal are paid more than workers who draw a low signal, because the former ones are more likely to be skilled. This wage scheme generates incentives to invest as the expected wage is higher for skilled workers. We assume that the investment cost follows a smooth distribution, and the condition that determines the equilibrium proportion of investors is that a worker invests if and only if the cost is below the expected wage increase. In addition, the wage schedule is affected by the relative price between the two goods. If the high-tech good becomes more expensive the individual incentives to acquire human capital increase.

The interaction between price effects and the informational externality creates a force in favor of specialization that, to our knowledge, is different from existing models of international trade. Equilibria emerge where, endogenously, countries specialize despite being fundamentally identical *ex ante*, that is, before human capital investment takes place. In these equilibria, workers from the country specializing in production of the high-tech good on average enjoy higher wages, but spend more resources on human capital investments than workers from the country specializing in the low-tech good.

To understand how asymmetric equilibria can emerge in our model, assume that the skill level in the foreign country increases. This has two effects in the foreign country. An

[1] In a previous version we considered a more general technology with one good being more intensive in skilled labor than the other. This generalization creates no additional qualitative insights.

immediate effect is that individual workers are more likely to be skilled, which may improve the incentives for skill acquisition.[2] In addition, the human capital increase changes the production possibilities set in a way so that the equilibrium production of the high-tech good increases, reducing its relative price and thereby reducing the wage differential between skilled and unskilled workers. This second effect, a pecuniary externality, affects both countries, but the first direct effect on the wage scheme only occurs in the country where the skill level is changed. Hence, wages are affected asymmetrically, which makes it possible that incentives for human capital investments go in opposite directions in the two countries.

Notice that the informational asymmetry is essential to generate endogenous specialization. The effects on relative prices from increasing relative abundance of skilled labor is the same as in a perfect information model. In a perfect information analogue of our model factor price equalization implies that incentives to acquire skills are independent of location, a property which falls apart when skills are observed with noise.

We demonstrate that there are parameterizations of the model in which the autarky equilibrium is unique, but where asymmetric equilibria arise under free trade. There is also always at least one symmetric equilibrium with no gainful trade that replicates the autarky allocation. Interestingly, the stability conditions in the autarky case differs from the free trade equilibrium, which is because the price effects from a given change in the proportion of skilled workers in a country are smaller when the market is bigger. As a result we find that opening up international trade may destabilize a unique and stable autarky equilibrium. Hence our model allows the possibility that cross country income differences is an inevitable aspect of free trade.

Rich countries are better off than poor countries, but this does not necessarily imply that the poor country is worse off under trade than in autarky. We show that it is possible that an asymmetric equilibrium Pareto dominates the autarky equilibrium. The intuition is that an increase in the skill level abroad may drive down the relative price of the high-tech good so much that exchanging the low-tech good for the high-tech good generates higher welfare in the poor country compared to domestic production.

There is no systematic advantage for large economies in our model. Equilibria of the two-country model can be reinterpreted as an equilibrium of a n country extension, where some countries specialize in the high-tech industry. Only the aggregate size of the part of the world

[2] The relationship between aggregate skills and incentives for skill acquisition is non-monotonic. In the extreme case in which all workers have high skills firms will ignore the noisy signal and pay all workers the value of the marginal product of a skilled worker, which completely eliminates any incentive to invest in human capital.

economy that specializes in the high-tech sector matters.[3] The model is thus consistent with a world where there is no particular relationship between size and development.

Our model is not the first to combine asymmetric information and international trade. Grossman and Maggi (2000) and Grossman (2004) have some elements that are similar to our setup. As in our setup, they consider a setup similar to Hecksher-Ohlin, but with skills being imperfectly observable. The difference is that they consider exogenous differences in the distribution of talent, while the crucial mechanism in our model is the interaction between skill distributions and incentives to acquire skills. We are aware of two papers considering asymmetric information and skill acquisition. Eicher (1999) considers a model that is significantly richer than ours in many ways, but the informational asymmetry is modeled in reduced form. Chisik (2003) develops a model where comparative advantages arise endogenously arise because of different countries coordinating on different equilibria. Also somewhat related is Araujo and Ornelas (2007), who analyze trade using a reputational model.

Park (2011) analyzes trade agreements under imperfect public monitoring, Zhang (2012) consider effects of asymmetric information when exporters are credit constrained, and Creane and Jeitschko show that weak institutions may result in welfare reducing trade in an adverse selection model. Razin and Sadka (2003) use an informational asymmetry to model the role of foreign direct investments, Casella and Rauch (2002) derive a role for minority groups in international trade using an informational friction, and McCalman (2002) considers the impact of asymmetric information in bargaining about trade agreements.

Another related paper is Costinot (2009) who, like us, seeks to formulate a more fundamental theory of comparative advantage. Indeed, the technology in Costinot (2009) is also based on the idea that human capital is more important for some firms than for others. The main difference is that the model ultimately derives cross country differences from exogenous differences in institutional quality and human capital. Our gains from specialization do not rely on any exogenous differences, which is the main difference between the papers. Here, one should also note that our results are robust to introducing exogenous productivity differences. If one country has a "fundamental" comparative advantage in the high-tech industry, it may still specialize as a low-tech industry as a result of the mechanism in our model, provided that the exogenous differences are small enough.

[3] However, size does affect stability conditions.

2 The Model

Two countries, labeled by $j = h, f$, are populated by a continuum of agents, where λ^h and $\lambda^f = 1 - \lambda^h$ denote the mass of agents in each country. Agents are price takers. We build on a simple $2 \times 2 \times 2$ trade model but with factors of production being workers with and without human capital. The model is closed by a stylized model of human capital acquisition and an informational technology borrowed from the statistical discrimination literature.[4]

All agents have identical preferences over the two goods, and each worker has an investment cost c drawn from interval $[\underline{c}, \overline{c}]$ according to a smooth cumulative density G. Investments are binary, the investment cost is independent of which country the agent lives in, and c is private information.[5] The utility of an agent c consuming the bundle (x_1, x_2) is $u(x_1, x_2) - c$ if the agent invests and $u(x_1, x_2)$ otherwise, where u is a homothetic and strictly quasi-concave function. We call workers who invest in human capital *qualified* and the others *unqualified.*

After the investments, nature assigns each worker a signal $\theta \in \{g, b\}$. For simplicity we assume that

$$\Pr[g|\text{worker qualified}] = \Pr[b|\text{worker unqualified}] = \eta > \frac{1}{2}, \tag{1}$$

where the restriction that $\eta > 1/2$ labels signals so that g is good news. Our preferred interpretation is that the unobservable investment decision is a costly effort decision and the signal is an imperfect measure of the costly effort, such as a test, a grade, or a letter of recommendation.

The two consumption goods are produced solely from qualified and unqualified labor, denoted q and n respectively, according to

$$y_1(q, n) = q; \quad y_2(q, n) = q + n. \tag{2}$$

All workers are thus perfect substitutes in industry 2, whereas only qualified workers contribute to the production of good 1[6]

[4] See Coate and Loury (1993) and Moro and Norman (2004)

[5] We will often assume that $\underline{c} < 0$. The rationale is that if an arbitrarily small fraction of workers like to make the investment even if there are no monetary gains, this eliminates "nuisance equilibria" with zero investments.

[6] This extreme technology is for simplicity only. Qualitatively, we need two sectors with different factor intensities, just like in the Heckscher-Ohlin model with fixed factor endowments.

3 Equilibrium

Our notion of equilibrium is analogous with a competitive equilibrium in a perfect information environment, but the informational asymmetry makes the treatment of the "labor supply" somewhat non-standard. For clarity, Section 3.1 therefore provides a detailed definition of equilibrium. We then show in Proposition 1 that, for *fixed* investments, versions of the welfare theorems hold: the equilibrium is characterized by a planning problem (where the informational asymmetry is built into the feasible set). This allows us to appeal to simple graphs in the analysis that follows.

3.1 Defining an Equilibrium

Consider an agent with realized wage w deciding on how to allocate her earnings between the two goods given prices $p = (p_1, p_2)$. The (ex post) maximized utility of the worker is

$$v(w,p) = \max_{x_1,x_2} u(x_1,x_2) \quad \text{subject to } p_1x_1 + p_2x_2 \leq w. \tag{3}$$

By strict quasi-concavity of $u(x_1,x_2)$, the optimization problem in (3) has a unique solution, and, with the usual notational abuse, we denote the demand functions by $x_1(w,p), x_2(w,p)$.

Firms cannot observe if a worker is qualified or not, so a labor demand is a map $l : \{g,b\} \to R_+$. Associated with any fraction of qualified workers, π, and a given labor demand l, the corresponding quantities of qualified and unqualified workers are

$$\begin{aligned} q &= l(g)\,\mu(g,\pi) + l(b)\,\mu(b,\pi) \\ n &= l(g)\,(1-\mu(g,\pi)) + l(b)\,(1-\mu(b,\pi)), \end{aligned} \tag{4}$$

where $\mu(\theta,\pi)$ denotes the posterior probability that a worker is qualified given prior π:

$$\mu(g,\pi) \equiv \frac{\eta\pi}{\eta\pi + (1-\eta)(1-\pi)} \qquad \mu(b,\pi) \equiv \frac{(1-\eta)\,\pi}{(1-\eta)\,\pi + \eta\,(1-\pi)}. \tag{5}$$

We assume that a strong law of large numbers applies and treat q and n in (4) as both expected and realized inputs of labor.

Without loss of generality there is a representative firm in each sector and each country, which takes a *wage schedule* $w^j : \{g,b\} \to R_+$ and output price p_i as given.[7] Using the

[7] The caveat is that the informational asymmetry would disappear if (qualified) workers could start their

production function (2) and (4), the profit maximization problem for a Sector 1 firm may be written as

$$\max_{l} p_1 \left(l\left(g\right)\mu\left(g,\pi^j\right)+l\left(b\right)\mu\left(b,\pi^j\right)\right)-w_g^j l\left(g\right)-w_b^j l\left(b\right), \tag{6}$$

where $\mu\left(\theta,\pi^j\right)$ is the posterior probability that a worker is qualified defined in (5). For sector 2, where qualified and unqualified workers are equally productive, the profit maximization problem is

$$\max_{l} p_2 \left(l\left(g\right)+l\left(b\right)\right)-w_g^j l\left(g\right)-w_b^j l\left(b\right). \tag{7}$$

Agents have rational expectations about the wages and prices, but face uncertainty about the realization of the signal. Denoting $v\left(w,p\right)$ the indirect utility function defined in (3), the expected utility for an agent in country j with investment cost c is

$$\eta v(w_g^j,p)+\left(1-\eta\right)v(w_b^j,p)-c \tag{8}$$

if a worker invest in human capital, and

$$\eta v(w_b^j,p)+\left(1-\eta\right)v(w_g^j,p) \tag{9}$$

if not. The worker is better off investing if and only if (8) exceeds (9), or if $c\leq\ (2\eta-1)\cdot(v(w_g^j,p)$ -$v(w_b^j,p))$. The implied proportion of investors in country j is thus

$$\pi^j=G\Big(\left(2\eta-1\right)\left(v(w_g^j,p)-v(w_b^j,p)\right)\Big). \tag{10}$$

To sum up: optimal consumption plans are defined in (3), the problems (6) and (7) describe the profit maximization problems for each sector, and (10) summarizes the individually optimal human capital investments.

What remains is to describe are the market clearing conditions. Factor market clearing requires that the aggregate demand for workers with each signal equals the mass of agents who draw the signal. That is, let $l_i^j=(l_i^j\left(g\right),l_i^j\left(b\right))$ be a labor demand scheme in industry

own firms. We rule this and other contractual solutions to the informational asymmetry out by assumption. One way to justify this is to assume that there is a minimum efficient scale for production and that only aggregate output, and not the performance of individual workers, can be observed.

j and country i and write the labor market clearing conditions as

$$\begin{aligned} l_1^j(g) + l_2^j(g) &= \eta \pi^j + (1-\eta)(1-\pi^j) \\ l_1^j(b) + l_2^j(b) &= (1-\eta)\pi^j + \eta(1-\pi^j). \end{aligned} \tag{11}$$

Finally, for the product market equilibrium conditions it is convenient to let x_i^j be the output in industry j and country i. That is

$$\begin{aligned} x_1^j &= l_1^j(g)\mu\left(g,\pi^j\right) + l_1^j(b)\mu\left(b,\pi^j\right) \\ x_2^j &= l_2^j(g) + l_2^j(b), \end{aligned} \tag{12}$$

which allows us to write the product market clearing conditions for the world market as

$$\sum_{j=h,f} \lambda^j \left(x_i^j - \underbrace{\left[\eta\pi^j + (1-\eta)(1-\pi^j)\right]}_{\#\text{agents with wage } w_g^j} x_i(w_g^j, p) - \underbrace{\left[(1-\eta)\pi^j + \eta(1-\pi^j)\right]}_{\#\text{agents with wage } w_b^j} x_i(w_b^j, p) \right) = 0 \tag{13}$$

Our definition of equilibrium is then:

Definition 1 *A Competitive Equilibrium consists of output prices p^*, wages w^{j*}, labor demands l_i^{j*}, outputs x_i^{j*}, and fractions of qualified workers π^{j*} for each country $j = h, f$ and industry $i = 1, 2$, satisfying:*

(1) l_1^{j*} solves (6) and l_2^{j*} solves (7) given $p_i = p_i^*$ and x_1^{j*} and x_2^{j*} are the associated profit maximizing outputs in $j = h, f$

(2) the product market clearing conditions in (13) are satisfied.

(3) the factor market clearing conditions in (11) are satisfied.

(4) π^{j*} satisfies (10) given $p = p^*$ and wages $w^j = w^{j*}$for $j = h, f$

3.2 The Production Possibilities Set

A useful way to represent the technology is in terms of the *production possibilities set.* The set of feasible production plans in a country depends on π and we let $X(\pi)$ denote the (per capita) production possibilities set in a country. The set $X(\pi)$ is depicted graphically in Figure 1. To understand the figure, first observe that $(x_1, x_2) = (0, 1)$ if all workers are producing good 2, and that $(x_1, x_2) = (\pi, 0)$ if all workers are producing good 1, because only a fraction π of the workers are productive in sector 1. Moreover, if all signal g workers

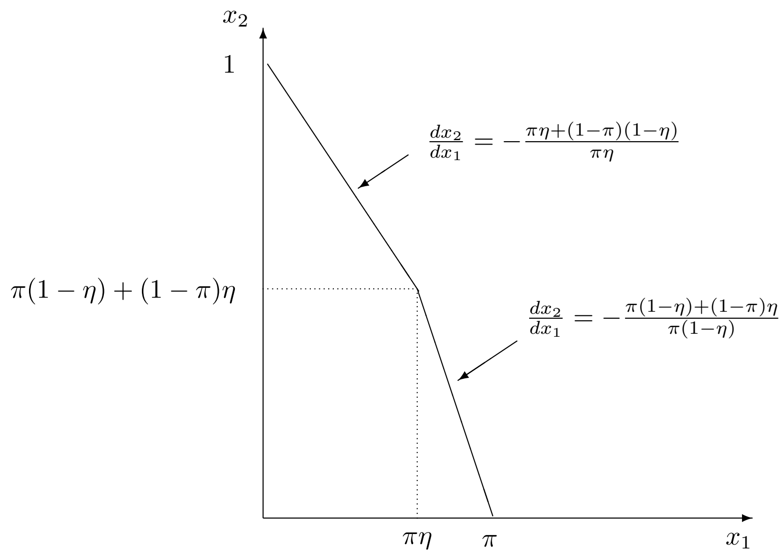

Figure 1: Per capita production possibilities in a country

are in sector 1 ($\pi\eta$ of these $\pi\eta + (1-\pi)(1-\eta)$ workers are productive) and all signal b workers (a total of $\pi(1-\eta) + (1-\pi)\eta$ such workers) are in sector 2, then the outputs are given by the point at the kink in the graph. The frontier to the right of the kink is steeper because in that region all g workers are employed in Sector 1, therefore to increase production firms must employ more b workers, who are less likely to be skilled. To the left of the kink instead, only g workers are employed in Sector 1.

The *world production possibilities* set is given by $X^W(\pi^h, \pi^f) = \lambda^h X(\pi^h) + \lambda^f X(\pi^f)$ and is convex by convexity of $X(\pi)$.

3.3 A Planning Characterization of Continuation Equilibria

We refer to a situation where all equilibrium conditions except the optimal investment condition (4) are fulfilled as a *continuation equilibrium.*[8] From the point of view an informationally unconstrained planner, a continuation equilibrium is inefficient: qualified and unqualified workers with the same signal are treated symmetrically, resulting in a misallocation of workers to jobs. However, if the symmetric treatment of workers with the same signal is viewed as a fundamental property of the environment, then the equilibrium allocation is (constrained) efficient *conditional on the investment behavior*. This allows us to

[8] This term is mainly due to lack of a better alternative. Due to the workers being non-atomic it does not make a difference whether investments are made before or simultanously with the wage posting.

describe aggregate equilibrium allocations as solutions to the planning problem,

$$\max_{(x_1,x_2)\in X^W\left(\pi^h,\pi^f\right)} u\left(x_1,x_2\right), \tag{14}$$

where $X^W\left(\pi^h,\pi^f\right)$ is defined in Section 3.2. This result allows us to appeal to intuitive graphs in the analysis that follows:

Proposition 1 *Suppose that $u\left(x_1,x_1\right)$ is homothetic. Then:*

1. *The aggregate world consumption in any continuation equilibrium is a solution to (14)*
2. *Suppose that $\left(x_1^*,x_2^*\right)$ solves (14), $\left(p_1^*,p_2^*\right)$ is a normal to a hyperplane that separates the set of bundles such that $u\left(x_1,x_2\right)\geq u\left(x_1^*,x_2^*\right)$ and $X^W\left(\pi^h,\pi^f\right)$, and that $w_g^{j*}=\max\left\{p_1^*\mu\left(g,\pi^j\right),p_2^*\right\}$ and $w_b^{j*}=\max\left\{p_1^*\mu\left(b,\pi^j\right),p_2^*\right\}$ in each country j. Then these prices, wages and aggregate consumptions are part of a continuation equilibrium.*[9]

The proof is in the appendix. Proposition 1 immediately implies:

Corollary 1 *Given any $\left(\pi^h,\pi^f\right)$ there is a unique continuation equilibrium up to a renormalization of the prices.*

4 A Parametric Specification

While the results presented below are more general, for simplicity of exposition in the remainder of the paper we will restrict attention to the case where

$$u\left(x_1,x_2\right)=x_1^{\alpha}x_2^{1-\alpha}. \tag{15}$$

Given the Cobb-Douglas preferences in (15) the demand functions are

$$x_1(p,w)=\frac{\alpha w}{p_1}\qquad x_2(p,w)=\frac{\left(1-\alpha\right)w}{p_2}. \tag{16}$$

Substituting back into (15) we obtain a expression for the continuation utility for a worker that earns wage w that is

$$v(w,p)=\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{p_1^{\alpha}p_2^{1-\alpha}}w. \tag{17}$$

[9] The allocation of workers in each country is somewhat complicated to describe in general, but is implicitly pinned down as the (almost always) unique worker allocation that can produce the equilibrium bundle.

We normalize by setting $p_2 = 1$ and, with some further abuse of notation, write $p(\pi), w_g^j(\pi)$ and $w_b^j(\pi)$ for the unique continuation equilibrium prices and wages in good 2 units, where $\pi = (\pi^h, \pi^f)$.

A qualified worker earns $w_g^j(\pi)$ with probability η and $w_b^j(\pi)$ with probability $1-\eta$. Symmetrically, an unqualified worker earns $w_g^j(\pi)$ with probability $1-\eta$ and $w_b^j(\pi)$ with probability η. Computing the expectation of $v(w,p)$ in (17) *conditional on investment* and subtracting from this the expectation of $v(w,p)$ *conditional on not investing* we get the *gross benefits of investment* for an agent in country j, denoted $B^j(\pi)$, which is given by

$$\begin{aligned} B^j(\pi) &= E\{v(w,p)\,|\text{qualified}\} - E\{v(w,p)\,|\text{unqualified}\} \\ &= \frac{(2\eta-1)(w_g^j(\pi) - w_b^j(\pi))}{(p(\pi))^\alpha}\alpha^\alpha(1-\alpha)^{1-\alpha}. \end{aligned} \tag{18}$$

Using condition 4 in Definition 1 we see that any π such that $\pi^j = G(B^j(\pi))$ for $j = h, f$ gives an equilibrium fraction of investors in each country. All that remains to calculate full equilibria is to derive expressions for the continuation equilibrium prices.

4.1 Continuation Equilibria in Autarky

As a benchmark, we first consider a closed economy. Suppressing the country index, we write π for the proportion of qualified workers. There are three possible types of continuation equilibria[10], illustrated in Figure 2. This is a somewhat unfortunate aspect of having only 2 signals. With a continuum of signals we would get a strictly convex production possibilities set and the tangency condition would determine a unique threshold signal. However, as it is much simpler to compute explicit examples with two signals we decided to stick with the more inelegant case.

Type A equilibria (allocation of workers "according to signals") Diagrammatically, this type occurs when the tangency is at the kink of the feasible set. That is, all workers with signal b (g) are working in the low (high) tech sector. Outputs are then $x_1 = \eta\pi$ and $x_2 = (1-\eta)\pi + \eta(1-\pi)$, so the demands in (16) pin down the price of the high-tech good as

$$p(\pi) = \frac{\alpha}{1-\alpha}\frac{(1-\eta)\pi + \eta(1-\pi)}{\eta\pi}. \tag{19}$$

[10] Calculations are straightforward but may be tedious. We provided more detailed steps in the web appendix Moro and Norman (2015).

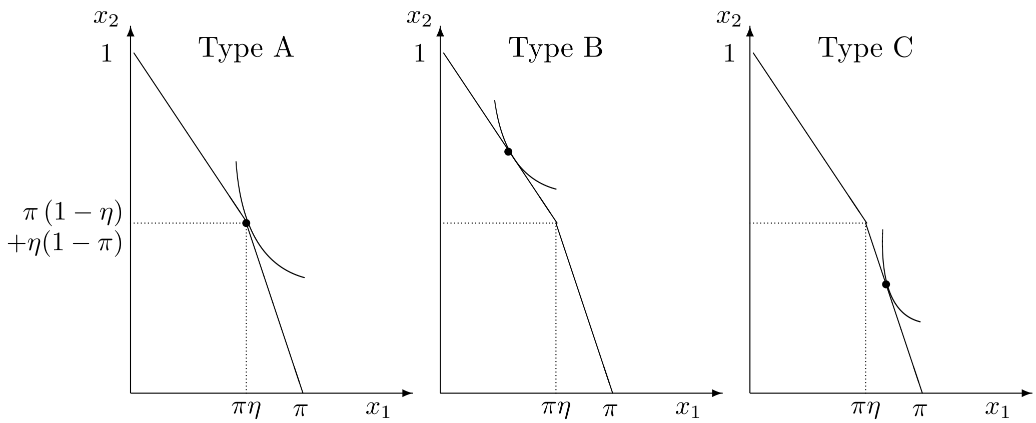

Figure 2: Three types of continuation equilibria

Candidate equilibrium wages are obtained by observing that zero profits is necessary for profit maximization. Since $p_2 = 1$, this immediately gives $w_b(\pi) = 1$. The high-tech firm sells $\eta\pi$ units at price $p(\pi)$ and hires $\eta\pi + (1-\eta)(1-\pi)$ workers with signal g. Zero profits Sector 1 therefore implies that

$$w_g(\pi) = p(\pi)\frac{\pi\eta}{\pi\eta + (1-\eta)(1-\pi)} = p(\pi)\,\mu(g,\pi)\,, \tag{20}$$

which has the interpretation that the wage equals the expected value of output. Finally, we have to check that a high-tech firm has no incentive to hire a worker with signal b, and that a low-tech firm has no incentive to hire a worker with signal g. These conditions give rise to inequalities that determine the region where a Type A continuation equilibrium exists (see Figure 3)

Type B equilibria (mixing of good signals) In Figure 2, this corresponds to a tangency to the left of the kink. Some workers with signal g work in Sector 2 (a fraction $1-\gamma$, defined in the equation below). These workers earn the same wage as in Sector 1, and, since all workers in the low-tech sector are paid 1, it follows immediately that $w_g(\pi) = w_b(\pi) = 1$. All that remains is therefore to determine the region where this is an equilibrium. To do this, one first observes that, for the high tech firm to make a zero profit, it must be that $p(\pi) = 1/\mu(g,\pi)$. The price of the high-tech good in units of the low tech good is thus determined on the "supply side" in this case. The tangency condition from the planning problem therefore determines the *outputs* that consumers are willing to purchase at these

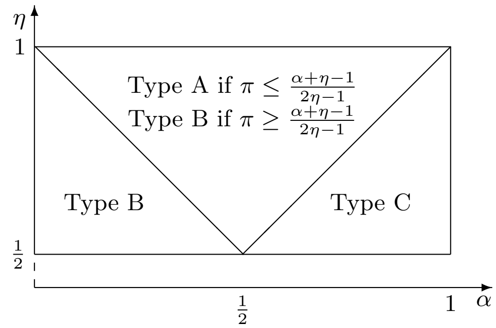

Figure 3: Types of autarky equilibria in the (α, η) space

prices, so this type of equilibrium requires that there exists $\gamma \in (0, 1]$ such that

$$\frac{\alpha}{(1-\alpha)\,p(\pi)} = \frac{\overbrace{\gamma\eta\pi}^{x_1}}{\underbrace{(1-\gamma)\left(\eta\pi + (1-\eta)(1-\pi)\right)}_{x_2 \text{ produced by } g\text{-workers}} + \underbrace{(1-\eta)\,\pi + \eta\,(1-\pi)}_{x_2 \text{ produced by } b\text{-workers}}}. \tag{21}$$

Type C equilibria (mixing of bad signals) This occurs if and only if $\alpha > \eta$, that is when the demand for the high-tech good is strong. In this case, some workers with signal b work in sector 1. Since no example that follow has an autarky equilibrium of this type we refer the reader to Moro and Norman (2015) for details.

4.2 Equilibrium investments in Autarky

A closed form expression for the incentives to invest as a function of π is obtained by substituting the wages and prices derived in Section 4.1 into (18). If $\alpha \le \eta$, this function may be written as[11],

$$B(\pi) = \Phi \max\left\{(2\eta-1)\left(\frac{\pi\eta}{\pi(1-\eta)+(1-\pi)\,\eta}\right)^{\alpha}\left(\frac{\alpha-(\pi\eta+(1-\pi)(1-\eta))}{\pi\eta+(1-\pi)(1-\eta)}\right), 0\right\}, \tag{22}$$

where $\Phi = \alpha^{\alpha}(1-\alpha)^{1-\alpha}$. Figure 4 plots $B(\pi)$ for two different sets of parameter values. One can show analytically that $B(\pi)$ is single-peaked, but not necessarily concave (see the example to the right). Any π such that $\pi = G(B(\pi))$ is an equilibrium fraction of investors. Since $G(B(\pi))$ is continuous and takes values on $[0, 1]$, existence follows trivially. The fixed point condition is illustrated in Figure 5, where $\eta = 2/3$, $\alpha = 1/2$ and G is uniform over $[\underline{c}, \overline{c}]$, with $\overline{c} - \underline{c} = 0.2$. Changes in $\underline{c}$ correspond to shifts in the cost distribution. If $\underline{c} < 0$

[11] See the web appendix for a detailed derivation, Moro and Norman (2015).

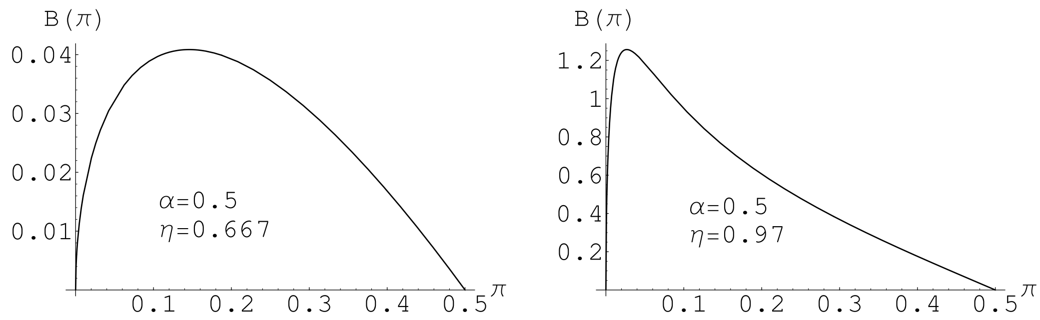

Figure 4: Gross incentives to invest under autarchy

(in case some workers prefer to invest at a zero wage difference) the equilibrium is unique. For $\underline{c} = 0$, there is a trivial equilibrium with no investments and an equilibrium with $\pi > 0$. As $\underline{c}$ gets slightly larger there are three equilibria, whereas if $\underline{c}$ is sufficiently large, only an equilibrium with no investment remains.

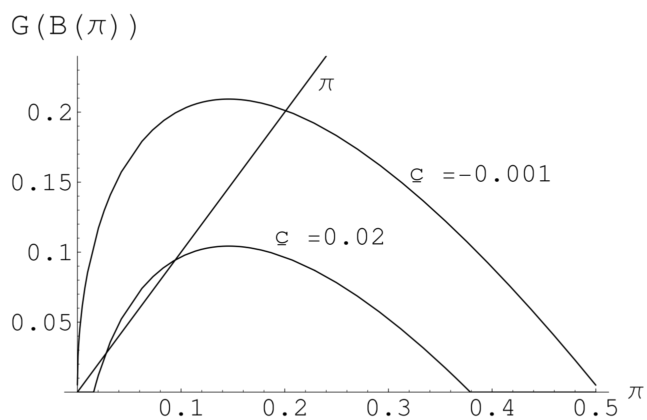

Figure 5: Equilibrium fixed point maps for two values of $\underline{c}$, with $\eta = 2/3, \alpha = 1/2$

4.3 Uniqueness of Autarky Equilibria

A useful feature of this parametrization is that there are simple sufficient conditions for when the autarky equilibrium is unique. While not being of much interest in itself, this result facilitates comparisons between trade and autarky.[12]

Proposition 2 *If $G(\cdot)$ is concave and $\underline{c} < 0$, then there is a unique autarky equilibrium.*

[12] With multiple autarky equilibria we would either have to make an equilibrium selection or make set-wise comparisons, which would obscure the economics of the model.

The interpretation of the condition $\underline{c} < 0$ is that the investment in itself provides utility to some (arbitrarily small proportion of) workers. This condition arises because the proof exploits that $G(B(\pi))$ cannot intersect the 45 degree line from below.

4.4 Equilibria in the Trade Regime

We now assume that h and f trade on a frictionless world market. The number of potential forms of continuation equilibria now swells to 9: in each country the allocation of workers may be like in any of the three types of autarky equilibria (however, mixing in both countries is a knife-edge possibility). To reduce the number of cases we therefore set $\eta = 2/3$, $\alpha = 1/2$, and $\lambda^h = \lambda^f = 1/2$ in the analysis that follows. With these parameter values the continuation equilibrium can be of three different forms. If countries are labeled so that $\pi^h \leq \pi^f$ the possibilities are [13]:

Type	A^T	B^T	C^T
$p(\pi^h, \pi^f)$	$\frac{4-\pi^f-\pi^h}{2(\pi^f+\pi^h)}$	$\frac{1+\pi^h}{2\pi^h}$	$\frac{2-\pi^f}{\pi^f}$
$w_g^h(\pi^h, \pi^f)$	$p(\pi^h, \pi^f)\frac{2\pi^h}{1+\pi^h}$	1	1
$w_b^h(\pi^h, \pi^f)$	1	1	1
$w_g^f(\pi^h, \pi^f)$	$p(\pi^h, \pi^f)\frac{2\pi^f}{1+\pi^f}$	$p(\pi^h, \pi^f)\frac{2\pi^f}{1+\pi^f}$	$p(\pi^h, \pi^f)\frac{2\pi^f}{1+\pi^f}$
$w_b^h(\pi^h, \pi^f)$	1	1	1
Exists when	$\pi^h \leq \pi^f \leq \frac{\pi^h(3-2\pi^h)}{1+2\pi^h}$	$\frac{\pi^h(3-2\pi^h)}{1+2\pi^h} \leq \pi^f \leq \frac{4\pi^h}{1+3\pi^h}$	$\pi^f \geq \frac{4\pi^h}{1+3\pi^h}$

Table 1: Continuation equilibria under international trade

Type $\mathbf{A}^T$ Equilibria (according to signals in both countries). This is the obvious analogue to equilibria of Type A in the autarky model. If investments in both countries are close to the value when this occurs in autarky, the continuation equilibrium is of this form.

Type $\mathbf{B}^T$ Equilibria (according to signals in f, mixing of good signals in h). In analogy with Type B equilibria in autarky, the equilibrium price is then determined from an indifference condition in the allocation of workers with signal g in country h.

Type $\mathbf{C}^T$ Equilibria (mixing of bad signals in f, all in low skill sector in h). This is just like a Type C equilibria in autarky, with some exogenous extra output of the low-tech good.

[13] Again see the web appendix Moro and Norman (2015) for details.

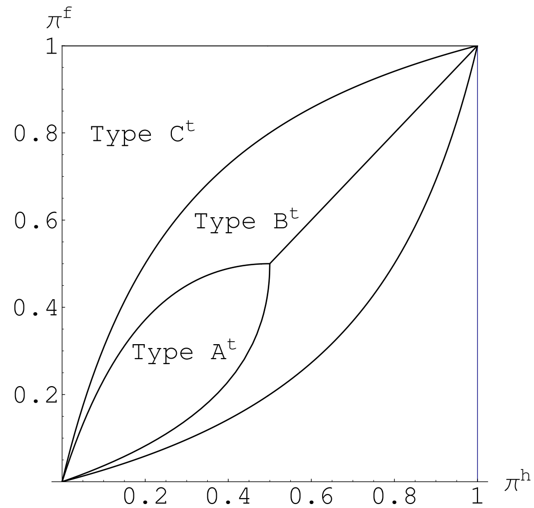

Figure 6: Types of asymmetric equilibria, with $\eta = 2/3, \alpha = 1/2$

The characterization of the relevant continuation equilibria is summarized in Table 1. It is understood that $\pi^h \leq \pi^f$, so Table 1 does provide a unique continuation equilibrium for any possible $(\pi) \neq (0,0)$ by reversing the roles of the countries when necessary. Figure 6 shows the regions of investment behavior where each type of equilibrium occurs.

The most instructive way to use the continuation equilibria in Table 1 and (18) is to express the incentives to invest for a worker in country f as

$$B^f(\pi) = \frac{1}{6}\left(\sqrt{p(\pi)}\mu\left(g, \pi^f\right) - \frac{1}{\sqrt{p(\pi)}}\right), \tag{23}$$

where $\mu(g, \pi) = 2\pi/(1+\pi)$ and

$$p(\pi) = \begin{cases} \frac{2-\pi^f}{\pi^f} & \pi^f \geq \frac{4\pi^h}{(1+3\pi^h)} \\ \frac{1+\pi^h}{2\pi^h} & \frac{\pi^h(3-2\pi^h)}{(1+2\pi^h)} \leq \pi^f \leq \frac{4\pi^h}{(1+3\pi^h)} \\ \frac{4-\pi^f-\pi^h}{2\left(\pi^f+\pi^h\right)} & \pi^h \leq \pi^f \leq \frac{\pi^h(3-2\pi^h)}{(1+2\pi^h)} \end{cases}. \tag{24}$$

Expression (23) shows that incentives are strictly increasing in the price of the high-tech good. Moreover, from (24), the equilibrium price is strictly decreasing in π^h in the range where some workers in h are in the high-tech sector. Hence, a decrease in investments at home improves incentives abroad and an increase in investments in the foreign country reduces incentives at home. In reduced form, this is like a negative cross-country externality

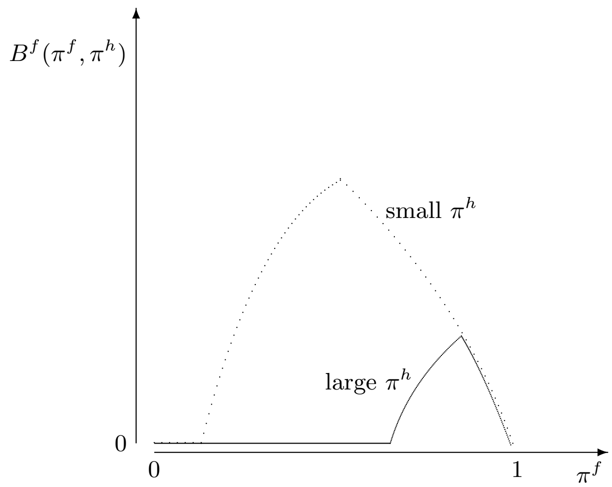

Figure 7: Incentives to invest in country f at different values of π^h

in human capital acquisition (Figure 7 shows how incentives in f are affected by π^h). These effects create equilibria where countries specialize: rich countries export the high-tech good and poor countries export the low-tech good, even when the autarky equilibrium is unique.

An Asymmetric Equilibrium May Be the Only Stable Outcome. A symmetric equilibrium, replicating autarky, always exists in the trade regime. However, for many parameterizations, this equilibrium is not stable when the economy is opened up for international trade.[14]

Assume $\underline{c} < 0$, so that there is a unique autarky equilibrium, denoted by π^A. Then π^A must be stable since $G(B(\pi))$ must intersect the 45^o line from above. It also follows that $(\pi) = \left(\pi^A, \pi^A\right)$ is an equilibrium when the countries are allowed to trade.

We want to analyze the effects of small deviations from the symmetric equilibrium. Consider the change in relative price first. When $\pi^h = \pi^f = \pi$ the price of the high-tech good is $p(\pi, \pi) = (4 - \pi - \pi)/2(\pi + \pi) = (2 - \pi)/\pi$. Differentiation of (24) gives

$$\begin{aligned} \frac{d}{d\pi} p(\pi, \pi) &= \frac{-1}{(\pi)^2} \qquad \text{(relevant under autarky)} \qquad (25) \\ \frac{\partial}{\partial \pi^f} p(\pi^h, \pi^f) &= \frac{-2}{(\pi^h + \pi^f)^2} \qquad \text{(relevant with trade).} \end{aligned}$$

[14]Since the model lacks real time, "stability" is a somewhat ad hoc criterion that corresponds to the adjustment dynamic where $\pi^j_{t+1} = G(B^j(\pi^j_t, \pi^k_t))$, $j, k = h, f$, $j \neq k$ (or the natural continuous analogue). Embedding the model in an OLG framework one obtains a dynamic system like this if one assumes that employers can not differentiate between workers of different cohorts.

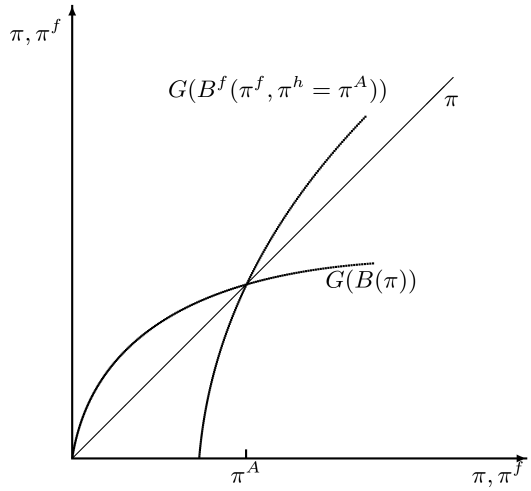

Figure 8: Best responses under trade and autarky, at the autarky equilibrium

Evaluating each expression at (π^A, π^A) we have that

$$\frac{d}{d\pi} p(\pi, \pi)\bigg|_{\pi=\pi^A} - \frac{\partial p(\pi^h, \pi^f)}{\partial \pi^f}\bigg|_{\pi^h=\pi^f=\pi^A} = \frac{-1}{(\pi^A)^2} - \frac{-2}{4\,(\pi^A)^2} = \frac{-1}{2\,(\pi^A)^2} < 0 \; . \tag{26}$$

An increase in investments thus have a larger negative impact on the price in autarky, as intuition would suggest. Autarky is equivalent to the trade regime with the added restriction that $\pi^h = \pi^f = \pi$. We can therefore use (23) for a comparison of the regimes. In the autarky case, we restrict the two arguments of B^f to be equal, while the second argument is unrestricted in the open economy case. Differentiating, we obtain (using p^A as shorthand notation for $p(\pi^A, \pi^A))$:

$$\frac{dB^j\,(\pi, \pi)}{d\pi}\bigg|_{\pi=\pi^A} = \underbrace{\frac{\sqrt{p^A}}{6} \frac{d\mu\,(g, \pi)}{d\pi}\bigg|_{\pi=\pi^A}}_{\text{"information effect"}} + \underbrace{\frac{1}{\sqrt{p^A}} \left(\mu\left(g, \pi^A\right) + \frac{1}{2p^A}\right) \frac{dp(\pi, \pi)}{d\pi}\bigg|_{\pi=\pi^A}}_{\text{"price effect"}} \tag{27}$$

$$\frac{\partial B^f\left(\pi^h, \pi^f\right)}{\partial \pi^f}\bigg|_{\substack{\pi^h=\pi^A \\ \pi^f=\pi^A}} = \underbrace{\frac{\sqrt{p^A}}{6} \frac{d\mu\left(g, \pi^f\right)}{d\pi^f}\bigg|_{\pi^f=\pi^A}}_{\text{"information effect"}} + \underbrace{\frac{1}{\sqrt{p^A}} \left(\mu\left(g, \pi^A\right) + \frac{1}{2p^A}\right) \frac{\partial p(\pi^h, \pi^f)}{\partial \pi^f}\bigg|_{\substack{\pi^h=\pi^A \\ \pi^f=\pi^A}}}_{\text{"price effect"}}, \tag{28}$$

In each case, the effect on incentives is decomposed as a positive "information effect" and a negative "price effect". The information effect in (27) is the same as in (28), but, by (26), the price effect is stronger in autarky, so the slope of $B^f\left(\pi^f, \pi^h = \pi^A\right)$ exceeds the slope of the autarky benefits of investment $B(\pi)$, when evaluating both functions at π^A (see Figure 8). Hence, it is possible that $G(B^f(\pi^f, \pi^h = \pi^A))$ intersects the 45^o line from below at $\pi^f = \pi^A$ even if $G(B(\pi))$ intersects from above. Since the curve $G(B^f(\pi^f, \pi^h = \pi^A))$ intersecting the 45^o line from below is a *sufficient* condition for local instability this shows that the autarky equilibrium may be destabilized by opening up for trade.[15]

Numerical Illustration of the existence of asymmetric equilibria. We turn now to the illustration of the main message of this paper, the existence of asymmetric equilbria in situations where there is a unique equilibrium without trade. We do so by construction, building on the setup presented so far in this section. Then, we will describe their welfare properties.

The simplest asymmetric equilibrium occurs when the poor country, which we label as country h, is fully specialized in the low-tech sector. In such an equilibrium, the wage gap in h is zero, so the fraction of qualified workers in h is pinned down as $\pi^h = G(0)$. Moreover, since the equilibrium under consideration must be either of type B^T or C^T, the calculation of the incentives in f is straightforward, and the proportion of qualified workers in f solves a single variable fixed point equation similarly to the autarky case, but with some extra production of x_2 performed in country h. Once π^f is obtained from this condition it only remains to check that firms in h have no incentives to hire workers signal g to produce the high-tech good.

Figure 9 illustrates how shifts in the cost distribution affect the possibility for asymmetric equilibria. The calculations assume a uniform G over $[\underline{c}, \underline{c} + 0.2]$ and $\underline{c}$ is treated as a variable and other parameters are held fix. The solid line represents equilibrium investments in country f if there were no incentives to invest in country h. The dotted line is the fraction that is willing to invest when the wage gap is zero, and the line in between represents equilibrium investments in autarky (or if symmetry is imposed). It cannot be seen in the figure, but it can be shown that $\pi = G(B(0))$ is a best response given that the other country invests in accordance with the solid line, so one country investing in accordance with the solid and the other in accordance with the dotted line is an equilibrium.

Both curves bend backwards, so there is a range with multiple equilibria also in the autarky model (when $\underline{c} \geq 0$). However, for $\underline{c} \in [-0.1, 0)$ there are asymmetric equilibria in the trade regime, and a unique autarky equilibrium. There is also a range to the right where

[15]Examples are easy to find. When c is uniformly distributed on $[0, 2]$, the unique (non-trivial) autarky equilibrium is $\pi = .0067$. The equilibrium where $\pi^f = \pi^h = 0.067$ is unstable under trade, while an asymmetric equilibrium with $\pi^f = .0283$, $\pi^h = 0$ is stable.

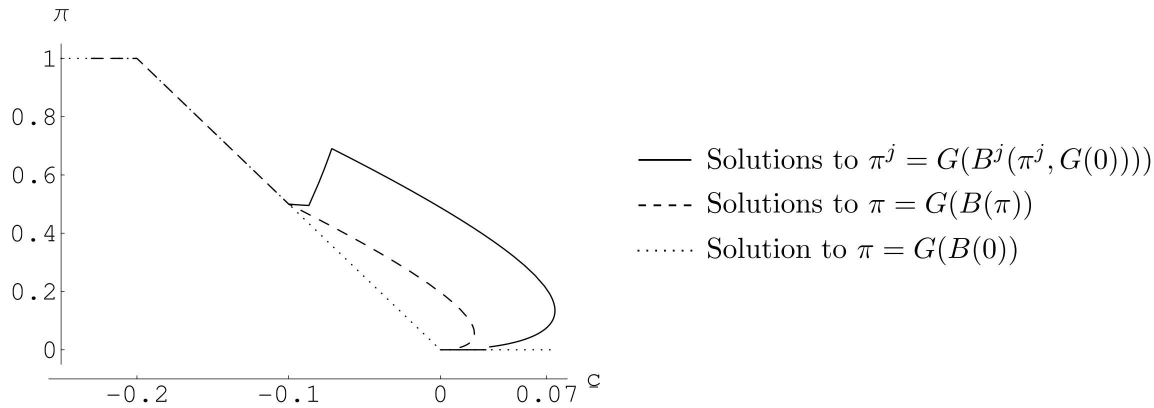

Figure 9: Equilibrium investments under trade with $\eta = 2/3, \alpha = 1/2$ for different values of $\underline{c}$.

there are non-trivial asymmetric trade equilibria, despite the unique autarky equilibrium being a trivial no investment equilibrium.

The following sections illustrate the welfare properties of the equililbria with trade.

4.4.1 Example 1: Specialization May be Beneficial Only to the Rich Country

Table 2 displays a parametrization where all country f citizens are better off in the asymmetric trade equilibrium than in the unique autarky equilibrium, and where all country h citizens are worse off in the asymmetric trade equilibrium than under autarky.[16]

Notice that the total world output of both goods is higher in the asymmetric equilibrium (see the second row of the table). While prohibitive trade barriers would make country h better off, it is also true that there exists transfer payments from f to h that can make both countries better off relative to the autarky equilibrium. Hence there are some productive gains from specialization despite the countries being funamentally identical.

4.4.2 Example 2: Specialization May Make Both Countries Better Off

We now consider an example where trade makes both countries better off. For maximal simplicity we rig this example so that the "free rider problem" in human capital investments

[16] Although some agents change their investment behavior in the comparison across equilibria, this does not complicate Pareto comparisons. The crucial fact is that (in the example) both qualified and unqualified workers gain (lose) in country f (h). All workers in the rich country have the option to invest as in the autarky equilibrium, so revealed preferences imply that all workers gain. Similarly, in the poor country all workers have the option to invest as in the trade equilibrium when in autarky, so again, by revealed preferences, all workers are better off in autarky.

$\eta = \frac{2}{3}, \alpha = \frac{1}{2}, c \sim U[-0.02, , 0.18]$	Trade, Country h	Trade, Country f	Autarky
Equilibrium Investment	$\pi^h = 0.1$	$\pi^f = 0.548$	$\pi = .269$
Per Capita Production	$y_1^h = 0$ $y_2^h = 1$	$y_1^f = 0.463$ $y_2^f = 0.226$	$y_1 = 0.179$ $y_2 = 0.577$
Per Capita Consumption	$x_1^h = 0.189$ $x_2^h = 0.5$	$x_1^f = 0.274$ $x_2^f = 0.726$	$x_1 = y_1$ $x_2 = y_2$
Gross incentives to invest	$B^h(\pi^h, \pi^f) = 0$	$B^f(\pi^h, \pi^f) = 0.090$	$B(\pi, \pi) = 0.034$
Gross expected utility	0.307	0.446	0.321
Expected utility net of inv. cost	0.308	0.427	0.319
Expected utility if invest	$0.307 - c$	$0.487 - c$	$0.346 - c$
Expected utility if don't invest	0.307	0.397	0.313
Wages	$w_g^h = 1$ $w_b^h = 1$	$w_g^f = 1.875$ $w_b^f = 1$	$w_g = 1.364$ $w_b = 1$
Expected Wage	1	1.452	1.154
Prices	$p_1 = 2.648$		$p_1 = 3.216$

Table 2: Trade and autarky equilibria in Example 1

is so severe the unique equilibrium under autarky is the trivial equilibrium. However, with trade, the existence of the other country means that, for any investment π^f in country f, the price of good 1 is higher than without trade under the assumption that there is no human capital investment in the other country. Hence, trade allows a new market to emerge that would not operate without trade.

In Table 3 we summarize one such example where the market for good 1 can only operate with international trade. Here, there are actually multiple trade equilibria and the numbers in the table is for the equilibrium with the largest fraction of investors in the country producing good 1.[17] Consumers are happier when consuming both goods than when consuming only one good. Because a new market opens up, trade is beneficial for both countries.

Pareto Improving Inequality. The example presented above is extreme, but specialization through trade may more generally be viewed as an imperfect "solution" to the informational problem in the model[18]. In the example, there is no way for a market to open unless the rewards for getting into the market are large enough. These rewards are bigger if only one country enters the market: the same "kick" from the local informational externality is generated at a smaller negative price effect. Specialization thus reduces the problem of under investment in human capital.

[17] There is also an equilibrium with $\pi^h = 0$, $\pi^f = 0.0157$. However, unlike the equilibrium in Table 3 this is unstable.

[18] For a detailed elaboration on this point in the context of discrimination, see Norman (2003).

$\eta = \frac{2}{3}, \alpha = \frac{1}{2}, c \sim U[.04, .24]$	Trade, Country h	Trade, Country f	Autarky
Equilibrium Investment	$\pi^h = 0$	$\pi^f = 0.353$	$\pi = 0$
Production	$y_1^h = 0$ $y_2^h = 1$	$y_1^f = 0.284$ $y_2^f = 0.323$	$y_1 = 0$ $y_2 = 1$
Consumption	$x_1^h = 0.107$ $x_2^h = 0.5$	$x_1^f = 0.177$ $x_2^f = 0.823$	$x_1 = y_1$ $x_2 = y_2$
Gross incentives to invest	$B^h(\pi^h, \pi^f) = 0$	$B^f(\pi^h, \pi^f) = 0.111$	$B(\pi, \pi) = 0$
Gross average utility	0.232	0.381	0
Avg. utility net of inv. cost	0.232	0.355	0
Expected utility if invest	$0.232 - c$	$0.452 - c$	$0 - c$
Expected utility if don't invest	0.232	0.342	0
Wages	$w_g^h = 1$ $w_b^h = 1$	$w_g^f = 2.433$ $w_b^h = 1$	$w_g = -$ $w_b = 1$
Expected Wage	1	1.647	1
Prices	$p_1 = 4.660$		$p_1 = -$

Table 3: Trade and autarky equilibria in Example 2

Even in less extreme cases, both countries may gain from specializing. As is illustrated in Figure 10 it is *always* true that the production possibilities set expand when moving from a situation where both countries invest at the same rate to an asymmetric investment profile for a constant total quantity of investors in the world. In the figure, the frontier to the left with the kink at point A is some symmetric investment profile, whereas the frontiers with kinks at B and C corresponds with an asymmetric investment profile. Assuming that countries are of equal size, the total number of investors in the world is unchanged, but the world production possibilities set is nevertheless larger (the frontier with kinks at D, A and E in the graph to the right). To understand this, note that the efficient way of increasing x_1 starting from the vertical intercept is to first only use workers from the country with investments $\pi+k$ with good signals, so initially the slope of the world production possibilities set must be the same as the set to the left with kink at C. The graph is drawn for the case where it is better to use high-signal workers from the low investment country than low signal workers from the high investment country in sector 1, but the result is fully general.

4.5 The Irrelevance of Size

Since this is a general equilibrium model with large countries, changes of the relative size of the countries will in general affect the *asymmetric* equilibria due to price effects. The nature of such changes depends on the parametrization. For example, if the example in Section 4.4.2 is extended to allow for different country sizes there is a critical size such that the country must fully specialize in the low-tech industry if it exceeds this critical size.

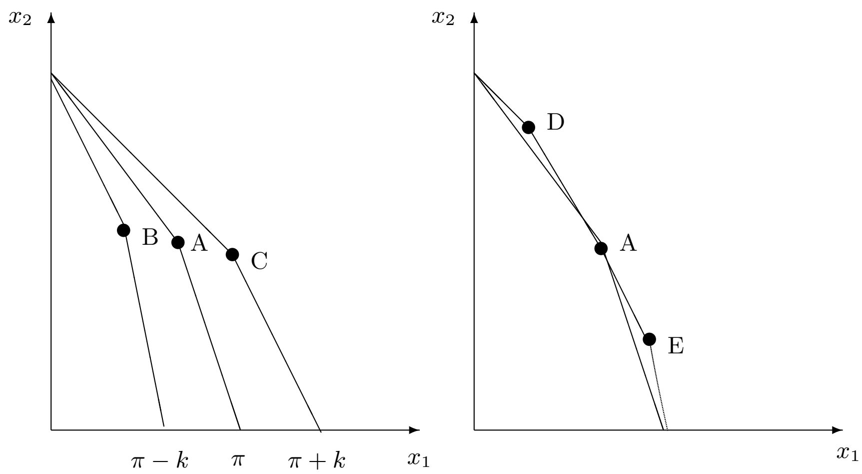

Figure 10: Specialization Expands the World Production Possibilities

Reducing the size of the country from $\frac{1}{2}$ on the other hand only improves incentives. Hence, there are circumstances where *the only asymmetric equilibrium* is that the small country becomes rich. It is also possible to set up examples that go the other way, where only the big country can end up on top (see Appendix A.3).

However, these scale effects are not really "country-scale-effects". Instead, we prefer to think of them as scale effects that have to do with the relative size of the North to the South. To understand this, suppose that there are n countries indexed by $j \in \{1, ..., n\}$. Let λ^j denote the size of country j and consider an equilibrium in this model where the set of countries is partitioned into the sets P and R and where $\pi^j = \pi^p$ for all $j \in P$ and $\pi^j = \pi^r$ for all $j \in R$. Finally let $\lambda^p = \sum_{j \in P} \lambda^j$ and $\lambda^r = \sum_{j \in R} \lambda^j$. This is an equilibrium if and only if (π^p, π^r) is an equilibrium in the two-country model with countries of sizes (λ^p, λ^r). There may of course be other equilibria as well, but at least for this form of equilibrium the size of the *individual country* is irrelevant and the relevant scale effect can be interpreted in our preferred manner.

A "development miracle" can be interpreted as a country that managees to re-coordinate from being part of the developing world to being part of the developed world. The model cannot explain how such a re-coordination is achieved, but, if the economy is small, the effects on the rest of the world are negligible. In contrast, a simultaneous re-coordination of a significant fraction of the "South" may lead to large enough relative price changes so that it is not worth the while as long as there is no change in the "North". Obviously, the model is too stylized for policy recommendations, but this nevertheless suggests that it may be misguided to use a few small successful countries as a model for all developing countries.

5 Concluding Remarks

This paper shows that it is possible to generate endogenous comparative advantages between identical countries in an essentially neoclassical model. Specialization and income differentials arise due to an informational externality: workers are better informed than firms about their abilities. This micro foundation implies that the scope of the externality is defined by barriers to labor mobility.

The two country model can be reinterpreted as equilibria of a n country model where countries cluster in two groups in terms of level of development. Equilibria of this model are neutral with respect to the size of individual countries, so the model is consistent with a world with no particular relationship between size and the level of development.

A natural extension is to introduce physical capital into the production technology. We believe that this would be interesting for analyzing the role of foreign capital and capital flight from poor countries. As this paper is about the qualitative effects of specialization due to asymmetric information about skills we have chosen to ignore physical capital. However, if capital and human capital are complementary in production, this would reinforce the effects analyzed in this paper.

To understand this, suppose initially that capital cannot flow between countries. The model is then (except for a capital market equilibrium condition) more or less the same as the model without capital. Consider an asymmetric equilibrium under the assumption that initial capital endowments are identical. As capital is more useful in the high-tech industry the return on capital would be higher in the rich country, so, with free capital mobility, it must be that the rich country has a higher per capita level of capital. More importantly for the mechanics of our model is that the movement of capital from the poor to the rich country would affect incentives to invest positively in the rich country and negatively in the poor country, strengthening the incentives to specialize.[19]

[19] Details are available on request from the authors.

A Appendix

A.1 Proof of Proposition 1

Proof. (Part 1) Consider an arbitrary equilibrium. Let $x^* = (x_1^*, x_2^*)$ denote the world production, where $x_i^* = \lambda^h x_i^{h*} + \lambda^f x_i^{f*}$ and x_i^{j*} denotes the production of good i in country j in equilibrium. Also let $l_i^{j*}(\theta)$ denote the corresponding input of workers with signal g in economy j and sector i. By profit maximization $p_i^* x_i^{j*} - \sum_{\theta \in g,b} w_\theta^{j*} l_i^{j*}(\theta) \geq p_i^* x_i^{j\prime} - \sum_{\theta \in g,b} w_\theta^{j*} l_i^{j\prime}(\theta)$ for any alternative plan $(x_i^{j\prime}, l_i^{j\prime}(\cdot))$. Adding over the two sectors and imposing the market clearing conditions on the labor market we conclude that $\sum_{i=1,2} p_i^* x_i^{j*} - w_g^{j*}\left(\eta\pi^j + (1-\eta)(1-\pi^j)\right) - w_b^{j*}\left((1-\eta)\pi^j + \eta(1-\pi^j)\right) \geq \sum_{i=1,2} p_i^* x_i^{j\prime} - w_g^{j*}\left(l_1^j(g) + l_2^j(g)\right) - w_b^{j*}\left(l_1^j(b) + l_2^j(b)\right)$ for all possible alternative production plans (feasible as well as non-feasible in the aggregate). Now for any feasible alternative allocation $l_1^j(g) + l_2^j(g) \leq \eta\pi^j + (1-\eta)(1-\pi^j)$ and $l_1^j(b) + l_2^j(b) \leq (1-\eta)\pi^j + \eta(1-\pi^j)$, implying that $\sum_{i=1,2} p_i^* x_i^{j*} \geq \sum_{i=1,2} p_i^* x_i^{j\prime}$ for any feasible alternative $(x_1^{j\prime}, x_2^{j\prime})$. Since this must hold in each country we conclude that $p^* x^* \geq p^* x'$ for any alternative feasible world production vector $x' = (x_1', x_2')$. Moreover, in order for (x_1^{j*}, x_2^{j*}) to be profit maximizing it must be that $\sum_{i=1,2} p_i^* x_i^{j*} - w_g^{j*}\left(\eta\pi^j + (1-\eta)(1-\pi^j)\right) + w_b^{j*}\left((1-\eta)\pi^j + \eta(1-\pi^j)\right) = 0$. Finally, since u is homothetic it follows from standard arguments that if $(x_1^{j*}(w), x_2^{j*}(w))$ solves the utility maximization problem for a worker with income w , then $(\frac{w'}{w} x_1^{j*}(w), \frac{w'}{w} x_2^{j*}(w))$ solves the utility maximization problem for a worker with income w'. Consider the program

$$\begin{aligned} &\max_{x_1,x_2} u(x_1, x_2) \\ \text{s.t} \quad & p_1^* x_1 + p_2^* x_2 \leq p_1^* x_1^* + p_2^* x_2^* = p_1^* \sum_{j=h,f} \lambda^j x_1^{j*} + p_2^* \sum_{j=h,f} \lambda^j x_2^{j*}, \end{aligned} \tag{A1}$$

where the star-superscript refers to equilibrium variables. The aggregate consumption bundle of any equilibrium must be a solution to (A1) because the problem gets the relative consumptions of x_1 and x_2 right and that $p_1^* x_1^* + p_2^* x_2^*$ is the aggregate world income. We thus conclude that if x^* is an equilibrium world consumption plan it must solve (A1). Since the set $X^W(\pi^h, \pi^f)$ is contained in the "budget set" of representative and $x^* \in X^W(\pi^h, \pi^f)$ it follows that x^* must be a solution to (14).

(Part 2) Let x^* solve (14) and let $V = \left\{x \in R_+^2 | u(x) > u(x^*)\right\}$. Quasi-concavity implies that V is a convex set. The set $X^W(\pi^h, \pi^f)$ is also convex (see Page 9). Moreover, $V \cap X^W(\pi^h, \pi^f) = \emptyset$, so the separating hyperplane theorem (Theorem 11.3. in Rockafellar

(1997)) implies that there exists some p^* such that $p^*x \geq p^*x^*$ for all $x \in V$ and $p^*x \leq p^*x^*$ for every $x \in X^W\left(\pi^h, \pi^f\right)$. Let the wages be given by $w_g^{j*} = \max\left\{p_1^*\mu\left(g, \pi^j\right), p_2^*\right\}$ and $w_b^{j*} = \max\left\{p_1^*\mu\left(b, \pi^j\right), p_2^*\right\}$, and let the allocation of workers be as in the planning solution. Observe in particular that if $p_1^*\mu\left(\theta, \pi^j\right) > p_2^*$, then no worker with signal θ is employed in sector 2 in the allocation that produces x^*. This is most easily seen in the differentiable case, where the optimality condition to (14) implies that $\frac{\partial u(x^*)}{\partial x_1^*} / \frac{\partial u(x^*)}{\partial x_2^*} = \frac{p_1^*}{p_2^*} > \frac{1}{\mu(g,\pi^j)}$. But, $\frac{1}{\mu(\theta,\pi^j)}$ is the cost of producing an extra unit of good 1 by giving up some country j workers with good signal currently in production of good 2, so we conclude that as if representative consumer would be better off if some of these workers would be switched to the production of good 1, contradicting optimality of x^* if $p_1^*\mu\left(\theta, \pi^j\right) > p_2^*$ and some of the j workers are assigned to sector 2. A symmetric argument holds for when the inequality is reversed. Hence, if $l_1^{*j}(\theta) > 0$, then $p_1^*\mu\left(\theta, \pi^j\right) = \max\left\{p_1^*\mu\left(\theta, \pi^j\right), p_2^*\right\} = w_\theta^{j*}$, implying that the profit from hiring any quantity workers with signal θ is zero in sector 1, whereas if $l_1^{*j}(\theta) = 0$, then $p_1^*\mu\left(\theta, \pi^j\right) \leq \max\left\{p_1^*\mu\left(\theta, \pi^j\right), p_2^*\right\} = w_\theta^{j*}$, so no gain can be earned from hiring a positive quantity. The argument for sector 2 is symmetric, which leads us to conclude that the outputs and (implicit) allocation of workers in the solution to (14) are consistent with profit maximizing behavior given the prices and wages constructed. ■

A.2 Proof of Proposition 2

Proof. We only prove the result for the case with $\alpha \leq \eta$. The case with $\alpha > \eta$ proceeds along the same lines, but the calculations are different. We first consider a uniform cost distribution and then generalize to concave distributions by use of a linear approximation. Since $G(B(0)) > 0$ by the assumption that $\underline{c} < 0$ and since $G(B(\pi)) > G(B(1)) = G(B(0))$ for all $\pi \in (0,1)$ an equilibrium $\pi^* > 0$ must exist. If $\pi^* = 1$ uniqueness is trivial (then $G(B(\pi)) > G(B(1)) \geq 1$ for all $\pi < 1$, implying that there is no other equilibrium) so we assume $\pi^* < 1$ in any equilibrium. For brevity we let $\phi = 2\eta - 1 > 0$ and define $b(\pi) \equiv \left(\frac{\pi}{\eta - \phi\pi}\right)^\alpha \left(\frac{\alpha}{\phi\pi + (1-\eta)} - 1\right)$. By straightforward algebra on the expression in (18) on can check that $B(\pi) = \Phi\phi\eta^\alpha b(\pi)$ for all π such that $B(\pi) \geq 0$. With c distributed uniformly on $[\underline{c}, \overline{c}]$ we then have that $G(B(\pi)) = Qb(\pi) + L$ for some positive constants Q and L and any $B(\pi) \in [\underline{c}, \overline{c}]$.[20] By a direct calculation we have that

$$b'(\pi) = \alpha \left(\frac{\pi}{\eta - \phi\pi}\right)^{\alpha-1} \left(\frac{\alpha}{\phi\pi + (1-\eta)} - 1\right) \frac{\eta}{(\eta - \phi\pi)^2} - \left(\frac{\pi}{\eta - \phi\pi}\right)^\alpha \frac{\phi\alpha}{(\phi\pi + (1-\eta))^2}$$

[20] Where in terms of the parameters of the model, $Q = \frac{1}{\overline{k} - \underline{k}}\Phi\phi\eta^\alpha$ and $L = -\frac{\underline{k}}{\overline{k} - \underline{k}} > 0$ given that $\underline{c} < 0$.

$$= b(\pi)\,\alpha\left(\frac{\eta}{\pi(\eta-\phi\pi)} - \frac{\phi}{(\phi\pi+(1-\eta))(\alpha-\phi\pi-(1-\eta))}\right). \tag{A2}$$

A sufficient condition for uniqueness is that $\frac{d}{d\pi}G(B(\pi^*)) < 1$ in any equilibrium π^*. We will show by direct computation that this holds in any equilibrium. We drop the $*$-superscript and note that for any equilibrium π (satisfying $\pi = G(B(\pi)) = Qb(\pi) + L$) we have:

$$\frac{d}{d\pi}G(B(\pi)) = Qb'(\pi) = Qb(\pi)\,\alpha\left(\frac{\eta}{\pi(\eta-\phi\pi)} - \frac{\phi}{(\phi\pi+(1-\eta))(\alpha-\phi\pi-(1-\eta))}\right) \tag{A3}$$

$$\begin{aligned} &= \alpha(\pi-L)\left(\frac{\eta}{\pi(\eta-\phi\pi)} - \frac{\phi}{(\phi\pi+(1-\eta))(\alpha-\phi\pi-(1-\eta))}\right) \\ &< \alpha\pi\left(\frac{\eta}{\pi(\eta-\phi\pi)} - \frac{\phi}{(\phi\pi+(1-\eta))(\alpha-\phi\pi-(1-\eta))}\right), \end{aligned}$$

where the equality comes from the fact that we are evaluating the derivative at an equilibrium point and the inequality since $L > 0$ and the bracketed expression needs to be strictly positive for $B(\pi) > 0$. But, the expression is increasing in α and $\alpha \leq \eta$, so

$$\alpha\pi\left(\frac{\eta}{\pi(\eta-\phi\pi)} - \frac{\phi}{(\phi\pi+(1-\eta))(\alpha-\phi\pi-(1-\eta))}\right) \tag{A4}$$

$$\leq \eta\pi\left(\frac{\eta}{\pi(\eta-\phi\pi)} - \frac{\phi}{(\phi\pi+(1-\eta))(\eta-\phi\pi-(1-\eta))}\right)$$

$$\begin{aligned} &= \frac{\eta^2}{(\eta-\phi\pi)} - \frac{\eta\pi}{(\phi\pi+(1-\eta))(1-\pi)} \quad \Big/ \quad \phi\pi+(1-\eta) < \phi+(1-\eta) = 2\eta-1+1-\eta = \eta \;\Rightarrow\; \tfrac{1}{\phi\pi+(1-\eta)} > \tfrac{1}{\eta} \quad \Big/ \\ &< \frac{\eta^2}{(\eta-\phi\pi)} - \frac{\pi}{(1-\pi)} = \frac{\eta^2}{(\eta-(2\eta-1)\pi)} - \frac{\pi}{(1-\pi)}, \end{aligned}$$

We now observe that $\frac{d}{d\eta}\left(\frac{\eta^2}{(\eta-(2\eta-1)\pi)}\right) = \frac{\eta(\eta+2\pi(1-\eta))}{(\eta-(2\eta-1)\pi)^2} > 0$, since $0 < \eta, \pi < 1$. Thus

$$\frac{\eta^2}{(\eta-(2\eta-1)\pi)} - \frac{\pi}{(1-\pi)} \leq \frac{1}{(1-\pi)} - \frac{\pi}{(1-\pi)} = \frac{1-\pi}{1-\pi} = 1. \tag{A5}$$

Combing (A4) and (A5) we get $\alpha\pi\left(\frac{\eta}{\pi(\eta-\phi\pi)} - \frac{\phi}{(\phi\pi+(1-\eta))(\alpha-\phi\pi-(1-\eta))}\right) < 1$, which with (A3) establishes the claim when C is a uniform distribution. For a general concave distribution note that for any $c > \underline{c}$ there exists some $c^* \in [\underline{c}, c]$ such that $G(c) = G'(c^*)\,c$. Hence the equilibrium must satisfy $\pi = G(B(\pi)) = G'(c^*)B(\pi)$ for some $c^* \leq B(\pi)$. Since G is concave it follows that $G'(B(\pi)) \leq G'(c^*)$, so $\frac{d}{d\pi}G(B(\pi)) = G'(B(\pi))B'(\pi) \leq G'(c^*)B'(\pi) = G'(c^*)\phi\eta^{\alpha}b'(\pi) = Qb'(\pi)$, for $Q = G'(c^*)\phi\eta^{\alpha} > 0$. The proof proceeds as with a uniform distribution with $L = 0$ ■

A.3 Scale Effects in the Two Country Model

We construct examples showing that scale effects may go either way. One way is to look at the extreme case where λ^h is near zero. Equilibria can be calculated by solving two separate (different) one-dimensional fixed point problems. Consider the incentives to invest in a country with fraction of investors π under the "small open economy" assumption that the price (of good 1) is fixed at p. Equilibrium wages in the small open economy are determined to generate zero profits: $w_g^O = \max\{p\mu(g,\pi), 1\}$ and $w_g^O = \max\{p\mu(b,\pi), 1\}$. The incentive to invest in the small open economy, denoted $B^O(\pi; p)$, is thus (using (18)),

$$B^O(\pi; p) = \frac{(2\eta-1)\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{p^{\alpha}} \max\{p\mu(g,\pi) - \max\{p\mu(b,\pi), 1\}, 0\}. \tag{A6}$$

If p^A is well defined (i.e., whenever the autarky equilibrium is non-trivial), then

$$\begin{aligned} B^h\left(\pi^h, \pi^A\right) &\to B^O\left(\pi^h, p = p^A\right) \text{ for all } \pi^h \in [0,1] \text{ as } \lambda^h \to 0 \\ B^f\left(\pi^h, \pi^A\right) &\to B\left(\pi^f\right) \text{ for all } \pi^f \in [0,1] \text{ as } \lambda^h \to 0, \end{aligned} \tag{A7}$$

Assume parameters implying a unique autarky equilibrium, and call it π^A. Let p^A denote the associated autarky price. If $\pi = \pi^A$, then $B^O\left(\pi^A; p = p^A\right) = B\left(\pi^A\right)$, and π^A solves

$$\pi = G\left(B^O\left(\pi; p = p^A\right)\right). \tag{A8}$$

While both (A8) and the autarky fixed point equation have π^A as a common solution, incentives diverge for other π since in autarky benefit price change as π changes whereas there are no such price effects in (A6). Equation (A8) will therefore in many cases have solutions different from π^A. Now, if π^O solves (A8) and if $\frac{d}{d\pi}\big|_{\pi=\pi^A}[\pi - G(B(\pi))] \neq 0$ and $\frac{d}{d\pi}\big|_{\pi=\pi^O}\left[\pi - G\left(B^O\left(\pi; p = p^A\right)\right)\right] \neq 0$, then, for λ^h small enough, there exists an equilibrium $\left(\pi^{h*}, \pi^{f*}\right)$ in the trade model near $\left(\pi^O, \pi^A\right)$.[21]

[21] The slope condition for the autarky equilibrium is satisfied under the conditions of Proposition 2. Its role is that if the equilibrium was at a tangency with the 45^0 line, the slightest effect from abroad could eliminate the equilibrium.

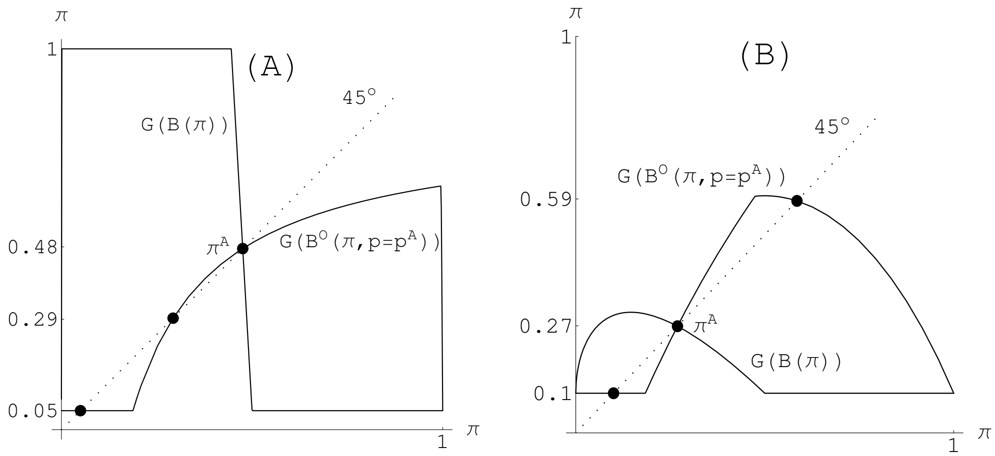

Figure 11: Equilibrium fixed point maps in a small open economy

We computed two examples to show that scale effects are indeterminate. Figure 11 (A) (computed using $\alpha = 1/2$, $\eta = .97$, $\underline{c} = -0.005$, $\bar{c} = .995$) illustrates the case where only the big country can be rich. There is a unique symmetric equilibrium at $\pi^A = 0.48$; (A6) intersects the 45^0 line only below π^A. There are two asymmetric equilibria where the big country invests at $\pi^h = \pi^A$ and the small country at $\pi^f = 0.05$ or 0.29. Figure 11 (B) was computed with the parameters as in Numerical Example 1. Both $\left(\pi^h, \pi^f\right) = (.27, 0.1)$ and $\left(\pi^h, \pi^f\right) = (.27, 0.59)$ are equilibria, so the small country can be either richer or poorer than the big country. Finally, when the unique autarky equilibrium is at $\pi^A = 0$, if the large country is large enough only the small country can be richer. Taken together, these three cases imply that there may be scale effects in favor of either the larger or the smaller economy, and that sometimes the equilibrium selection matters.

References

Araujo, Luis F. and Emanuel Ornelas, "Trust-based trade," *mimeo, available at SSRN 642803*, 2007.

Casella, Alessandra and James E Rauch, "Anonymous market and group ties in international trade," *Journal of International Economics*, October 2002, *58* (1), 19–47.

Chisik, Richard, "Export industry policy and reputational comparative advantage," *Journal of International Economics*, 2003, *59* (2), 423–451.

Coate, Stephen and Glenn C. Loury, "Will affirmative-action policies eliminate negative stereotypes?," *The American Economic Review*, December 1993, *83* (5), 1220–1240.

Costinot, Arnaud, "On the origins of comparative advantage," *Journal of International Economics*, 2009, *77* (2), 255–264.

Eicher, Theo S., "Training, adverse selection and appropriate technology: Development and growth in a small open economy," *Journal of Economic Dynamics and Control*, April 1999, *23* (5), 727–746.

Grossman, Gene M., "The distribution of talent and the pattern and consequences of international trade," *Journal of Political Economy*, 2004, *112* (1), 209–239.

_ **and Giovanni Maggi**, "Trade and Diversity," *The American Economic Review*, February 2000, *90* (5).

McCalman, Phillip, "Multi-lateral trade negotiations and the most favored nation clause," *Journal of International Economics*, 2002, *57* (1), 151–176.

Moro, Andrea and Peter Norman, "A general equilibrium model of statistical discrimination," *Journal of Economic Theory*, 2004, *114* (1), 1–30.

_ **and** _ , "Endogenous Comparative Advantage Web Appendix," *mimeo*, 2015, *available upon request or from the authors' web page.*

Norman, Peter, "Statistical discrimination and efficiency," *The Review of Economic Studies*, 2003, *70* (3), 615–627.

Park, Jee-Hyeong, "Enforcing international trade agreements with imperfect private monitoring," *The Review of Economic Studies*, 2011, *78* (3), 1102–1134.

Razin, Assaf and Efraim Sadka, "Gains from FDI inflows with incomplete information," *Economics Letters*, 2003, *78* (1), 71–77.

Rockafellar, R. Tyrrell, "Convex Analysis. Princeton landmarks in mathematics," 1997.

Zhang, Xin, "Credit Constraints and Decisions in Exports: Theory under Asymmetric Information," *Journal of Mathematical Finance*, 2012, *2* (03), 238.