Wage Gaps and Test Score Differences: Incentives or Pre-Market Factors?

Andrea Moro (joint with Peter Norman)

Federal Reserve Board, Washington DC, January 27, 2005

The question

- Empirically disentangle two different sources of racial wage inequality

1. Pre-market factors (Neal and Johnson, JPE 96)
2. Incentives to acquire human capital

- Horse-race between the two hypotheses using a model of statistical discrimination that nests the two explanations.
The question

- Empirically disentangle two different sources of racial wage inequality
  1. Pre-market factors (Neal and Johnson, JPE 96)
  2. Incentives to acquire human capital

- Horse-race between the two hypotheses using a model of statistical discrimination that nests the two explanations.

- Other explanations: racism (Bowlus & Eckstein 2002), initial conditions/catching up
Facts

- Black/white wage gap has been roughly stable since the 80s (problem for the “catching up” hypothesis)

\[
\ln(wages), \text{NLSY males, 1991} \\
\text{Black } - 0.244 (0.026) \\
\text{AFQT } - 0.367 (0.012)
\]

Neal & Johnson argues that this is evidence against statistical discrimination.
Facts

• Black/white wage gap has been roughly stable since the 80s (problem for the “catching up” hypothesis)

• Much of the difference is “explained” by differences in “human-capital-like” variables such as schooling and test-scores
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\[
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<table>
<thead>
<tr>
<th></th>
<th>Black</th>
<th>AFQT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>(-0.244) (0.026)</td>
<td>(-0.072) (0.027)</td>
</tr>
<tr>
<td>AFQT</td>
<td></td>
<td>0.172 (0.012)</td>
</tr>
</tbody>
</table>
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- Black/white wage gap has been roughly stable since the 80s (problem for the “catching up” hypothesis)

- Much of the difference is “explained” by differences in “human-capital-like” variables such as schooling and test-scores

- “Returns to AFQT” the same for blacks and whites. Neal & Johnson argues that this is evidence against statistical discrimination.
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\[
\ln(wages), \text{ NLSY males, 1991}
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<table>
<thead>
<tr>
<th></th>
<th>Blacks</th>
<th>Whites</th>
</tr>
</thead>
<tbody>
<tr>
<td>AFQT</td>
<td>0.208 (0.031)</td>
<td>0.183 (0.017)</td>
</tr>
</tbody>
</table>
The standard argument

- We can observe measures of skill (e.g. AFQT)

- If minorities appear to have lower returns to skill, they have less incentives to invest in skills

- Test: look at difference in returns to skill between groups: if they are insignificant, then statistical discrimination is rejected

Problem with the argument

- Measures of skill are not perfectly correlated with market valued skills.

- Presumably, the econometrician cannot observe the same signals that employers observe.

- Using a different signal introduces an “error in variable” bias.
A simple model to illustrate the problem

- Human capital investment $h \in \{0, 1\}$

- Cost of investment, worker $i \sim G$ is $C(i) = i$

- Workers with human capital are called qualified and produce 1; unqualified produce 0

- Competitive firms observe only a noisy signal of productivity $z \in \{\text{good, bad}\}$
A simple model to illustrate the problem

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<table>
<thead>
<tr>
<th>Type of worker</th>
<th>Probability of obtaining ( z = \text{good} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qualified</td>
<td>( pq )</td>
</tr>
<tr>
<td>Unqualified</td>
<td>( pu &lt; pq )</td>
</tr>
</tbody>
</table>

\[ pu < pq \]
Incentives to invest in human capital

- $\pi^J =$ proportion of people who invest in group $J$

- Firms pay expected productivity, computed using Bayes’ rule:

  \[
  w^J(\text{good}; \pi^J) = \frac{\pi^J pq}{\left(1 - \pi^J\right) pu + \pi^J pq}
  \]

  \[
  w^J(\text{bad}; \pi^J) = \frac{\pi^J (1 - pq)}{\pi^J (1 - pq) + (1 - \pi^J)(1 - pu)}
  \]
Incentives to invest in human capital

• $\pi^J = \text{proportion of people who invest in group } J$

• Firms pay expected productivity, computed using Bayes’ rule:

$$w^J(\text{good}; \pi^J) = \frac{\pi^J p_q}{(1 - \pi^J) p_u + \pi^J p_q}$$

$$w^J(\text{bad}; \pi^J) = \frac{\pi^J (1 - p_q)}{\pi^J (1 - p_q) + (1 - \pi^J) (1 - p_u)}$$

• Incentives to invest :

$$I(\pi^J) = E_z[w^J(z; \pi^J)|\text{invest}] - E[w^J(z; \pi^J)|\text{don’t}]$$

$$= (p_q - p_u) [w^J(\text{good}; \pi^J) - w^J(\text{bad}; \pi^J)]$$
Incentives to invest in human capital

Example drawn using $pq = .8$ and $pu = .5$. 

\[ \Pi_B 0.5 1 \Pi_W 0.8 \Pi \]
Equilibrium

Example drawn using $p_q = .8$ and $p_u = .5$.

Using an appropriate distribution of costs $G$ (thick line) we can support a pair of equilibrium fraction of investors $\pi^B < \pi^W$ so that

$$\pi^J = G(I(\pi^J))$$
The econometric problem

Consider an econometrician observing \( x \in \{HIGH, LOW\} \), independent from the firms’ signal

<table>
<thead>
<tr>
<th>Type of worker</th>
<th>Probability of obtaining ( x = HIGH )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Qualified</td>
<td>( r_q )</td>
</tr>
<tr>
<td>Unqualified</td>
<td>( r_u &lt; r_q )</td>
</tr>
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Returns to the observable measure of skill

The econometrician can measure:

\[ R^J(\pi^J) = E[w^J|HIGH] - E[w^J|LOW] \]

Proposition: 

\[ R^J(\pi^J) < I^I(\pi^J) \]
Returns to the observable measure of skill

The econometrician can measure:

\[ R_J^J(\pi^J) = E[w^J|HIGH] - E[w^J|LOW] \]

Proposition: \[ R_J^J(\pi^J) < I_I^I(\pi^J) \]

Intuition:

*HIGH* workers may have “firm’s signal” *good* or *bad*.

\[ \implies E[w^J|HIGH] < w^J(\text{good}) \]

*LOW* workers may have firm’s signal *good*, or *bad*,

\[ \implies E[w^J|LOW] > w^J(\text{bad}) \]
The possibility of an erroneous conclusion

The bias depends on $\pi$.

Using $pq = 0.8, pu = 0.5$, $r_q = 0.8, r_u = 0.1$.

Firm’s signal

Econometrician’s signal

Note: the econometrician’s signal is more informative signal than firm’s signal.
The possibility of an erroneous conclusion

The bias depends on $\pi$.

Using $p_q = 0.8$, $p_u = 0.5$, $r_q = 0.8$, $r_u = 0.1$

Firm’s signal

Econometrician’s signal

Note: the econometrician’s signal is more informative signal than firms'signal
The Model to be Estimated

- Continuous human capital $h$. Cost of $h$ is $C(h, k) = h/k$, $\ln(k) = N(\mu_k, \sigma_k)$

- Signal observed by firms: $z = \ln(h) + \varepsilon$, $\varepsilon \sim N(0, \sigma_\varepsilon^2)$

- Preferences $u(w, h) = \ln w - c(h, k)$

- Competitive firms $w(z) = E[h|z]$
Looking for a Log-Normal Equilibrium

Assume log normal $h$ (later verify this is the case) with mean/var. $\mu_h, \sigma_h^2$

\[ z = \ln(h) + \varepsilon \implies f(\ln(h)|z) = N\left(\mu_h \frac{\sigma_k^2}{\sigma^2 + \sigma_k^2} + z \frac{\sigma_k^2}{\sigma^2 + \sigma_k^2}, \left(\frac{\sigma_k^2 \sigma^2}{\sigma^2 + \sigma_k^2}\right)^2\right) \]

Wages are log-linear in $z$:

\[
\begin{align*}
  w(z) &= E(h|z) = \exp \left( \alpha \frac{\sigma^2}{\sigma^2 + \sigma_k^2} + z \frac{\sigma_k^2}{\sigma^2 + \sigma_k^2} + \frac{\alpha}{2 \sigma^2 + \sigma_k^2} \right) \\
  \ln(w) &= \alpha + \beta \cdot z
\end{align*}
\]


Workers’ problem

Expected utility linear in \( \ln(h) \):

\[
E_z \left[ \ln \left( w(z) \right) \mid h \right] = E_z \left[ \alpha + \beta z \mid h \right] = E_z \left[ \alpha + \beta (\ln(h) + \varepsilon) \mid h \right] = \alpha + \beta E_z (z \mid h) = \alpha + \beta \ln h
\]

\[
\max_{h \geq 0} \alpha + \beta \ln (h) - \frac{h}{k} \\
\implies h(k) = \beta k
\]

i.e. human capital is indeed lognormal, \( \ln(h) \sim N(\mu_k + \ln(\beta), \sigma^2_k) \)
Restriction imposed by the equilibrium

\text{Stdev of log } h : \sigma_h = \sigma_k

\text{Mean of log } h : \mu_h = \mu_k + \ln \beta = \mu_k + \ln \left( \frac{\sigma_h^2}{\sigma_e^2 + \sigma_h^2} \right)

= \mu_i + \ln \left( \frac{\sigma_i^2}{\sigma_e^2 + \sigma_i^2} \right).
Equilibrium

Model has unique log-normal equilibrium (generating log-linear wages).

For any \((\mu_k, \sigma_k, \sigma_\varepsilon)\) there is an equilibrium where

\[
\begin{align*}
h(k) &= \beta k \\
w(z) &= \exp(\alpha + \beta z),
\end{align*}
\]

where

\[
\begin{align*}
\alpha &\equiv \mu_h \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_k^2} + \frac{1}{2} \frac{\sigma_k^2 \sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \sigma_k^2} \\
\beta &\equiv \frac{\sigma_k^2}{\sigma_\varepsilon^2 + \sigma_k^2}.
\end{align*}
\]

In this equilibrium, \(\ln h \sim N \left(\mu_k + \ln \beta, \sigma_k^2\right)\).
The econometric problem

• Assume that the econometrician observes a proxy of skill

\[ x = \ln h + \delta, \]

where \( \delta \sim N \left( 0, \sigma^2_\delta \right) \) is assumed to be independent of \( \varepsilon \).

• Since \( \ln h = z - \varepsilon \) it follows immediately that

\[ x = z - \varepsilon + \delta, \]

which means that a standard OLS regression of wages on AFQT scores leads to a downwards biased estimate of \( \beta \) in the equilibrium wage function.
Error in variable bias

- $x = \ln(h) + \delta$: econometrician’s variable

- $z = \ln(h) + \varepsilon$: firms’ signal

- $\ln(w) = \alpha + \beta z = \alpha + \beta x + \beta(-\delta + \varepsilon)$

- The regressor ($x$) is correlated with the disturbance

$$\Rightarrow \lim(b_{LS}) = \beta \cdot \frac{\sigma_{\delta}^2}{\sigma_{\delta}^2 + \sigma_k^2} = \frac{\sigma_k^2}{\sigma_{\varepsilon}^2 + \sigma_k^2} \cdot \frac{\sigma_k^2}{\sigma_{\delta}^2 + \sigma_k^2}$$
## Data


<table>
<thead>
<tr>
<th></th>
<th>&lt; High Sc.</th>
<th>High Sc.</th>
<th>College or more</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Black</td>
<td>White</td>
<td>Black</td>
</tr>
<tr>
<td>Obs.</td>
<td>(75)</td>
<td>(109)</td>
<td>(323)</td>
</tr>
<tr>
<td>E[ln(wage)]</td>
<td>6.46</td>
<td>6.64</td>
<td>6.61</td>
</tr>
<tr>
<td>SD[ln(wage)]</td>
<td>0.33</td>
<td>0.41</td>
<td>0.44</td>
</tr>
<tr>
<td>E[AFQT]</td>
<td>-1.1</td>
<td>-0.71</td>
<td>-0.61</td>
</tr>
<tr>
<td>SD[AFQT]</td>
<td>0.51</td>
<td>0.69</td>
<td>0.73</td>
</tr>
<tr>
<td>Corr[wage,AFQT]</td>
<td>0.04</td>
<td>0.4</td>
<td>0.18</td>
</tr>
</tbody>
</table>

(full sample)
Identification strategy

- We observe AFQT, not x, therefore assume for some $C, D$:
  \[ C + D \cdot AFQT_i = \ln(h_i) + \delta_i \]

- Assume wages are observed with measurement error $u \sim N(0, \sigma_u^2)$

- Restrict some parameters to be identical across groups: $C, D, \sigma_\delta$

- Use restrictions implied by the model and its equilibrium
10 parameters to be estimated

- $\mu_B^k, \mu_W^k, \sigma_B^2, \sigma_W^2$: distributions of the investment cost
- $\sigma_B^2, \sigma_W^2$: the variance in firms’ signal
- $\sigma_u^2$: measurement error in wage data
- $C, D, \sigma_\delta^2$: scaling of AFQT and variance of scaled test
10 parameters to be estimated

- $\mu_k^B, \mu_k^W, \sigma_k^2B, \sigma_k^2W$: distributions of the investment cost
- $\sigma_{\varepsilon^2}^B, \sigma_{\varepsilon^2}^W$: the variance in firms' signal
- $\sigma_u^2$: measurement error in wage data
- $C, D, \sigma^2_\delta$: scaling of AFQT and variance of scaled test

We can then compute incentives using equilibrium restriction

$$\beta^J = \frac{\sigma_k^{J2}}{\sigma_{\varepsilon^2}^{J2} + \sigma_k^{J2}}, J = B, W$$
Identifying Conditions

\[
\begin{align*}
p \lim (b^J_{LS}) &= D \beta^J \frac{\sigma^2_J}{\sigma^2_J + \sigma^2_\delta} \\
\ln(E[w^J]) &= \mu^J_k + \ln(\beta^J) + \frac{\sigma^2_J}{2} \\
\text{Var}[\ln(w^J)] &= \beta^J \sigma^2_J + \sigma^2_u \\
C + D \cdot E[AFQT^J] &= \mu^J_k + \ln(\beta^J) \\
D^2 \text{VAR}[AFQT^J] &= \text{VAR}[\ln(h^J)] = \sigma^2_J + \sigma^2_\delta
\end{align*}
\]

10 conditions in 10 unknowns, but not all parameters are identified
What we can identify

<table>
<thead>
<tr>
<th>High School Sample</th>
<th>Estimates</th>
<th>Stderr</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>0.199</td>
<td>0.037</td>
</tr>
<tr>
<td>$\sigma_k^{2W} - \sigma_k^{2B}$</td>
<td>0.0023</td>
<td>0.0025</td>
</tr>
<tr>
<td>$\mu_k^W + \ln(\beta_k^W) - (\mu_k^B + \ln(\beta_k^B))$</td>
<td>0.189</td>
<td>0.036</td>
</tr>
<tr>
<td>$\beta_k^B \sigma_k^{2B}$</td>
<td>0.0113</td>
<td>0.0045</td>
</tr>
<tr>
<td>$\beta_k^W \sigma_k^{2W}$</td>
<td>0.0106</td>
<td>0.0037</td>
</tr>
<tr>
<td>$\sigma_u^{2B}$</td>
<td>0.178</td>
<td>0.022</td>
</tr>
<tr>
<td>$\sigma_u^{2W}$</td>
<td>0.147</td>
<td>0.011</td>
</tr>
</tbody>
</table>
Additional restrictions from the model

Use $\beta < 1$ and $\sigma_\delta > 0$ to provide an upper bound for $\sigma_k$ and a lower bound for $\beta$

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<tr>
<td>High School</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta^B$</td>
<td>0.532</td>
<td>0.210</td>
</tr>
<tr>
<td>$\beta^W$</td>
<td>0.452</td>
<td>0.171</td>
</tr>
<tr>
<td>&lt; High School</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta^B$</td>
<td>0.065</td>
<td>0.179</td>
</tr>
<tr>
<td>$\beta^W$</td>
<td>0.524</td>
<td>0.296</td>
</tr>
</tbody>
</table>
Scenarios, less than high school sample

Solid line: $\beta^W$
Dotted line $\beta^B(\beta^W)$
Scenarios, high school sample

Solid line: $\beta^W$
Dotted line $\beta^B(\beta^W)$
Conclusion

• A naive look at returns to observable (to the investigator's) may give us biased conclusions about the importance of statistical discrimination

• We look at the data with the guidance of the restrictions imposed by a formal equilibrium model

• Even if we don’t achieve full identification, we can provide some clues

• Preliminary results: black high school graduate are statistically discriminated against, but not black high school dropouts
The End
Statistical discrimination: a theory of self fulfilling stereotypes

Incomplete information is crucial.
Data, full sample

Full Sample

<table>
<thead>
<tr>
<th></th>
<th>Black</th>
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</tr>
</thead>
<tbody>
<tr>
<td>N. of obs.</td>
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<td>825</td>
</tr>
<tr>
<td>E[wage]</td>
<td>6.64</td>
<td>6.89</td>
</tr>
<tr>
<td>SD[wage]</td>
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<td>0.43</td>
</tr>
<tr>
<td>E[AFQT]</td>
<td>-0.57</td>
<td>0.44</td>
</tr>
<tr>
<td>SD[AFQT]</td>
<td>0.82</td>
<td>0.93</td>
</tr>
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<td>Corr[wage,AFQT]</td>
<td>0.34</td>
<td>0.38</td>
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